5.23 ⋆ A damped oscillator satisfies the equation (5.24), where $F_{dmp} = -b\dot{x}$ is the damping force. Find the rate of change of the energy $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$ (by straightforward differentiation), and, with the help of (5.24), show that dE/dt is (minus) the rate at which energy is dissipated by F_{dmp}.

$m\ddot{x} + b\dot{x} + kx = 0$

5.26 ⋆⋆ An undamped oscillator has period $\tau_o = 1.000$ s, but I now add a little damping so that its period changes to $\tau_1 = 1.001$ s. What is the damping factor β? By what factor will the amplitude of oscillation decrease after 10 cycles? Which effect of damping would be more noticeable, the change of period or the decrease of the amplitude?

5.28 ⋆⋆ A massless spring is hanging vertically and unloaded, from the ceiling. A mass is attached to the bottom end and released. How close to its final resting position is the mass after 1 second, given that it finally comes to rest 0.5 meters below the point of release and that the motion is critically damped?

3-12. A simple pendulum consists of a mass m suspended from a fixed point by a weightless, extensionless rod of length l. Obtain the equation of motion and, in the approximation that $\sin\theta \cong \theta$, show that the natural frequency is $\omega_0 = \sqrt{g/l}$, where g is the gravitational field strength. Discuss the motion in the event that the motion takes place in a viscous medium with retarding force $2m\sqrt{gl}\,\dot{\theta}$.

3-29. An R-L-C circuit (see Figure 3-18b) contains an inductor of 0.01 H and a resistor of 100 Ω. The oscillation frequency is 1 kHz. If at $t = 0$ the voltage across the capacitor is 10 V and the current is 0, find the current 0.2 ms later.

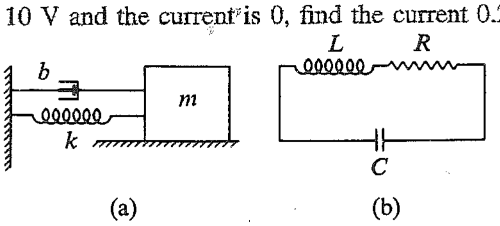

FIGURE 3-18

5.44 ⋆⋆ Another interpretation of the Q of a resonance comes from the following: Consider the motion of a driven damped oscillator after any transients have died out, and suppose that it is being driven close to resonance, so you can set $\omega = \omega_o$. (a) Show that the oscillator's total energy (kinetic plus potential) is $E = \frac{1}{2}m\omega^2A^2$. (b) Show that the energy ΔE_{dis} dissipated during one cycle by the damping force F_{dmp} is $2\pi m\beta\omega A^2$. (Remember that the rate at which a force does work is Fv.) (c) Hence show that Q is 2π times the ratio $E/\Delta E_{dis}$.

5.41 ⋆ We know that if the driving frequency ω is varied, the maximum response (A^2) of a driven damped oscillator occurs at $\omega \approx \omega_o$ (if the natural frequency is ω_o and the damping constant $\beta \ll \omega_o$). Show that A^2 is equal to half its maximum value when $\omega \approx \omega_o \pm \beta$, so that the full width at half maximum is just 2β. [*Hint:* Be careful with your approximations. For instance, it's fine to say $\omega + \omega_o \approx 2\omega_o$, but you certainly mustn't say $\omega - \omega_o \approx 0$.]

3-45. A grandfather clock has a pendulum length of 0.7 m and mass bob of 0.4 kg. A mass of 2 kg falls 0.8 m in seven days to keep the amplitude (from equilibrium) of the pendulum oscillation steady at 0.03 rad. What is the Q of the system?